

τ Anomalous Couplings and Radiation Zeros in the $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ process

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Abstract

The process $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ contains configurations of the four-momenta for which the scattering amplitude vanishes (Radiation Zeros or Null Zone). These Radiation Zeros only occur for couplings given by a gauge theory, in particular the standard model. Therefore they are sensitive to physics beyond the standard model as the anomalous magnetic and weak moment of the τ lepton. In this article we compute the effect of these anomalous interactions on the Radiation Zeros in the above mentioned process.

I. INTRODUCTION

The bounds on the anomalous magnetic moment of the τ lepton a_γ , are much weaker than the ones for electron and muon. Due to the τ lepton's short lifetime of $(291.0 \pm 1.5) \times 10^{-15}$ seg, its anomalous magnetic moment can not be measured by a spin precession method and no direct measurement of a_γ exists so far. Because of the impossibility of measuring a_γ by a spin precession method, the present bounds have been obtained by analysis of collision experiments. In that sense, interesting articles dealing with the computation of bounds for a_γ have been recently published. The OPAL Collaboration [1] uses a reaction proposed by A. Mendez and A. Grifolds [2] some years ago. They obtained limits on a_γ from the non-observation of anomalous $\tau\bar{\tau}\gamma$ production at LEP. In other article, G.A.Gonzales-Sprinberg, A.Santamaria and J.Vidal using LEP1, SLD, and LEP2 data for tau lepton production, and data from CDF, D0 and LEP2, for W-decays into tau lepton, established model independent limits on non-standard electromagnetic and weak magnetic moments of the tau lepton [3]. These obtained bounds are still far away from the theoretical precision for a_γ : $(1177.3 \pm 0.3) \times 10^{-6}$. This weakness of the a_γ bounds is unfortunate since large deviations from the S.M. values are expected for the τ lepton. In particular, in composite models one would expect larger effects for the tau lepton than for the rest of the leptons.

In this article we are interested in studying the effect of the anomalous magnetic and weak moment of the τ lepton on radiation zeros. In this respect we study the Radiation Zeros to arise in the $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ process at future e^+e^- collider energies. These kind of zeros are known as type 2 zeros [4]. Definitions and study of this kind of zeros is presented in section 2 in the soft photon approximation. This discussion should be viewed as a qualitative argument for the existence and the location of radiation zeros. Our purpose there is only to show, in a very simple fashion, where the radiation zeros are. When we numerically calculate the cross-section using a Monte Carlo technique in section 4 we do not make use of this approximation. There we calculate the cross section without approximation on the photon energy and we only impose cuts on the photon energy to ensure that the photon is the softest particle in the final state. In this conditions the numerical results for the radiation zeros positions should be similar to obtained in the soft photon approximation.

Following Ref [5], in order to analyze the tau magnetic and weak moments, we will use an effective lagrangian description. Thus, in section 3, we describe the effective lagrangian formalism. In section 4 we present the results for the zeros position, the standard model cross section and the effective coupling effects. Finally, in section 5 we give our conclusions.

II. RADIATION ZEROS

In certain high-energy scattering processes involving charged particles and the emission of one photon there are configurations of the final state particles for which the scattering amplitude vanishes. Such null zones are known as radiation zeros. Detailed study about this effect can be found in the literature for several process [4]. The process that we are studying in this paper ($e^+e^- \rightarrow \tau\bar{\tau}\gamma$) presents radiation zeros of type 2. For this kind of zeros is not necessary that the charged particles have same sing charges but all particles including the photon have to lie in the same plane. To have a first idea about the location of these zeros we present in this section a study within the soft photon approximation. The discussion is particularly simple when one considers the amplitude for the emission of a single soft photon.

The matrix elements for one soft photon emission can be written as

$$M_\gamma \propto J_\mu \epsilon^\mu M_0 \quad (1)$$

where M_0 is the leading order 4-fermion matrix element and ϵ^μ is the photon polarization. In the other hand the current J_μ is given by

$$J_\mu = \sum_i \alpha_i \frac{p_\mu^i}{p^i \cdot k} \quad (2)$$

where the sum run over the four charged fermions ($e, \bar{e}, \tau, \bar{\tau}$) with 4-momenta p_1, p_2, p_3 and p_4 respectively and k is the photon 4-momenta. The coefficient α_i is -1 for e and $\bar{\tau}$ and 1 for τ and \bar{e} .

In the soft photon approximation with massless fermions, we have $k^0 = \omega_\gamma$, $p_1^0 = p_2^0 = p_3^0 = p_4^0 = \sqrt{s}/2$. In this conditions, if we denote by \tilde{n}_i the directions for the spatial momentum of the charged fermions and by \tilde{n} the direction for the photon spatial momentum, then we have $\vec{p}_i = \frac{\sqrt{s}}{2}\tilde{n}_i$ and $\vec{k} = \omega_\gamma\tilde{n}$.

Using the 4-momentum conservation in the center of mass frame

$$\frac{\sqrt{s}}{2}\tilde{n}_3 + \frac{\sqrt{s}}{2}\tilde{n}_4 + \omega_\gamma\tilde{n} = 0 \quad (3)$$

we have the obvious condition that we call **condition 1** : The 3-momentum of the three final state particles (directions \tilde{n}_3, \tilde{n}_4 and \tilde{n}) lie in the same plane (Moreover, in the soft photon approximation where $\omega_\gamma \rightarrow 0$, we have $\tilde{n}_4 = -\tilde{n}_3$).

On the other hand the radiation zero condition implies that

$$M = 0 \quad \text{thus} \quad J^\mu \epsilon_\mu = 0 \quad (4)$$

Considering the expression for the current J_μ we have, according to gauge invariance, that

$$J^\mu k_\mu = \sum_{i=1,4} \alpha_i \frac{p_i^\mu}{p_i \cdot k} k_\mu = \sum_{i=1,4} \alpha_i = 0 \quad (5)$$

In the other hand by gauge invariance we have that $\epsilon_\mu k^\mu = 0$. In this conditions the transformation

$$J^\mu \rightarrow J^\mu + a k^\mu \quad (6)$$

$$\epsilon^\mu \rightarrow \epsilon^\mu + b k^\mu \quad (7)$$

with arbitrary a and b leaves M invariant.

Then we can choose the numbers a and b such that $\epsilon^\mu = (0, \vec{\epsilon})$ and $J^\mu = (0, \vec{J})$.

In this conditions, the Zero Radiation request implies $\vec{J} \cdot \vec{\epsilon} = 0$, but if we impose this condition for every photon polarization then the useful conditions read $\vec{J} = 0$.

To find the expression for \vec{J} we have first to find the constant a imposing that $J^0 = 0$, then

$$J^0 = \sum_{i=1,4} \alpha_i \frac{p_i^0}{p_i \cdot k} + a k^0 = 0 \quad (8)$$

If we call $x_i = \cos \theta_{i,\gamma}$, where $\theta_{i,\gamma}$ is the angle between the directions of the photon and the fermion i , then

$$a = \frac{2}{\omega_\gamma^2} \left(\frac{x_1}{1-x_1^2} - \frac{x_3}{1-x_3^2} \right), \quad (9)$$

where we have used $k^0 = \omega_\gamma$, $p_i^0 = \sqrt{s}/2$, $x_2 = -x_1$ and $x_4 = -x_3$ which is valid in the soft photon approximation.

With the a value found above we obtain the following expression for \vec{J}

$$\vec{J} = \frac{2}{\omega} \left[\frac{-1}{1-x_1^2} \tilde{n}_1 + \frac{1}{1-x_3^2} \tilde{n}_3 + \left(\frac{x_1}{1-x_1^2} - \frac{x_3}{1-x_3^2} \right) \tilde{n} \right] \quad (10)$$

which obviously satisfies $\vec{J} \cdot \vec{n} = 0$ (gauge invariance).

By using the Radiation Zero condition $\vec{J} = 0$ we have

$$\frac{-1}{1-x_1^2} \vec{n}_1 + \frac{1}{1-x_3^2} \vec{n}_3 + \left(\frac{x_1}{1-x_1^2} - \frac{x_3}{1-x_3^2} \right) \vec{n} = 0 \quad (11)$$

which implies that the vectors \vec{n}_1 , \vec{n}_3 and \vec{n} lie in same plane. That is we call the **condition 2**. Thus taking into account both conditions (**1** and **2**), we obtain the necessary condition for radiation zeros: the momenta of all the particles included the photon lie in the same plane. We call this the *planarity condition*. In order to find the exact position of the radiation zeros in the soft photon approximation, we project the vectorial equation (11) over the directions \vec{n}_1 and \vec{n}_3 . Calling $x_{31} = \cos \theta_{31}$, we obtain the equations

$$x_{31} - x_1 x_3 = 1 - x_3^2 \quad (12)$$

$$x_{31} - x_1 x_3 = 1 - x_1^2 \quad (13)$$

then $x_3 = \pm x_1$. Using the solution $x_3 = x_1$ in some of the above equations we obtain $x_{31} = 1$. This solution is uninteresting as all particles are in the beam axes. Considering now the solution $x_3 = -x_1$ and replacing it in the equations (12) or (13), we find a solution for the positions of the radiation zeros:

$$x_1 = \pm \frac{1}{\sqrt{2}} \sqrt{1 - x_{31}}, \quad (14)$$

or in function of the angles shown in Fig. 2 we obtain according to Ref. [4]

$$\theta_\gamma = \arccos \left[\pm \frac{1}{\sqrt{2}} \sqrt{1 - \cos \theta_{CM}} \right] \quad (15)$$

The above equation give us the radiation zeros positions. There are two zeros for each value of the angle between the outgoing τ and the beam axes, as we show in fig 3.

III. THE EFFECTIVE LAGRANGIAN APPROACH

In the last few years, effective lagrangians have been used as an adequate framework to study low energy effects of physics beyond the standard model (SM). Since the SM gives a very good description of all physics at the energies available at present accelerators, then one expects that any deviation of the SM can be parametrized by an effective lagrangian built with the same fields and symmetries that the SM. In this conditions, the effective lagrangian contains a renormalizable piece, the SM theory, and non-renormalizable operators of dimension higher than 4, which are suppressed by inverse powers of the high energy physics scale, Λ . The leading non-standard effects will come from the operators with the lowest dimension. Those are dimension six operators. In particular, there are only two six-dimension operators which contribute to the anomalous magnetic moments [6]:

$$\begin{aligned} \mathcal{O}_B &= \frac{g'}{2\Lambda^2} \bar{L}_L \phi \sigma_{\mu\nu} \tau_R B^{\mu\nu} \\ \mathcal{O}_W &= \frac{g}{2\Lambda^2} \bar{L}_L \vec{\tau} \phi \sigma_{\mu\nu} \tau_R \vec{W}^{\mu\nu} \end{aligned} \quad (16)$$

where $L_L = (\nu_L, \tau_L)$ is the tau leptonic doublet and ϕ is the Higgs doublet. $B^{\mu\nu}$ and $W^{\mu\nu}$ are the $U(1)_Y$ and $SU(2)_L$ field strength tensors, and g' and g are the corresponding gauge couplings. Thus, we write our effective Lagrangian as

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \alpha_B \mathcal{O}_B + \alpha_W \mathcal{O}_W + h.c. \quad (17)$$

As we are not interested in studying CP violation effects, then we will take the coupling α_B and α_W as real. Moreover, we will consider them as free parameters without any further assumption.

The interaction lagrangian can be written in terms of the physical fields A_μ , Z_μ and W_μ^\pm . In our particular case, we are only interested in the effective electromagnetic and neutral weak interaction, since we are studying a process which involves only interactions with γ and Z bosons. In this conditions, the relevant lagrangians is

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \epsilon_\gamma \frac{e}{2m_Z} \bar{\tau} \sigma_{\mu\nu} \tau F^{\mu\nu} + \epsilon_Z \frac{e}{2m_Z \sin \theta_W \cos \theta_W} \bar{\tau} \sigma_{\mu\nu} \tau Z^{\mu\nu} + \dots \quad (18)$$

where the dots represent non-relevant pieces of the lagrangian and $F_{\mu\nu}$ and $Z_{\mu\nu}$ are the electromagnetic and weak field strength tensor respectively. We have expressed the coupling in function of the parameters ϵ_γ and ϵ_Z . This parameters are functions of the constants a_γ and a_Z which are directly related to the experimental measurement and theoretical calculations:

$$\epsilon_\gamma = \frac{m_Z}{2m_\tau} a_\gamma \quad (19)$$

$$\epsilon_Z = \frac{m_Z \sin \theta_W \cos \theta_W}{2m_\tau} a_Z \quad (20)$$

$$(21)$$

IV. THE $E^+ E^- \rightarrow \tau \bar{\tau} \gamma$ PROCESS

In this section we study the $e^+ e^- \rightarrow \tau \bar{\tau} \gamma$ process which involves electro-weak interactions plus additional anomalous magnetic and weak moment couplings given by the operators in Eq. (16). The corresponding Feynman diagrams are shown in Fig.1. We do not present the amplitude explicitly here because it is a very long and not illuminating expression. The calculation was done in the helicity formalism [7] and the mass of the final leptons was considered as null due to the high center of mass energies considered. The black blob in Fig.1 represents the effective operator contributions. The kinematics of the process is represented in Fig. 2 where the relevant scattering angles are shown. The intensity of the effective vertices are measured by the parameters $\epsilon_{\gamma,Z}$. We take different and representative values of typical bounds founded by another authors [1–3]. To have a first filling of the effective vertices effects on the radiation zeros we plot in Fig. 4 the square amplitude as a function of θ_γ for $\theta_{CM} = 20^\circ$ and $\sqrt{s} = 1000$ GeV. We include (dotted line) the effective contribution for $\epsilon_\gamma = \epsilon_Z = 0.1$. This figure does not represent any measurable quantities but it is included here to show the effect of the anomalous couplings on the radiation zeros in the squared scattering amplitude. As we can see, the effective contribution destroys the standard model radiation zero and then became itself in a possible observable to bound anomalous effects over the τ interactions. However, in practice, experiments deal with binned quantities and a more realistic study should take these into account. In this conditions, we have done a Monte Carlo calculation of the cross section, where we have included realistic cuts to eliminate the collinear singularity and to impose the planarity condition for type 2 radiation zeros. The numerical integration was done using the numerical subroutine RAMBO [8] and considering three different center of mass energies and sets of cuts adequate for each one. The center of mass energies considered were 500, 1000 and 2000 GeV. The aim is to see whether the radiation zeros remain visible after a more realistic analysis. We generate a sample of $\tau \bar{\tau} \gamma$ events using a Monte Carlo which includes the exact phase space and matrix elements without approximation on the photon energy. The following sequence of cuts is applied for each center of mass energies:

$$\begin{aligned} \sqrt{s} = 500 \text{ GeV} & \rightarrow 10 \text{ GeV} < \omega_\gamma < 100 \text{ GeV} < E_{\bar{\tau}}, E_\tau \\ \sqrt{s} = 1000 \text{ GeV} & \rightarrow 10 \text{ GeV} < \omega_\gamma < 200 \text{ GeV} < E_{\bar{\tau}}, E_\tau \\ \sqrt{s} = 2000 \text{ GeV} & \rightarrow 10 \text{ GeV} < \omega_\gamma < 400 \text{ GeV} < E_{\bar{\tau}}, E_\tau \end{aligned} \quad (22)$$

These cuts ensure that the photon is the softest particle in the final state and then the positions of the radiation zeros are near to the ones obtained in the soft photon approximation. (Moreover it is required that the photon must be separated in angle from the beam and τ and $\bar{\tau}$ directions).

$$\theta_{\gamma,beam} > 5^0, \quad \theta_{\gamma,\tau} > 2^0, \quad , \theta_{\gamma,\bar{\tau}} > 2^0 \quad (23)$$

With this cuts we define a measurable sample of $\tau\bar{\tau}\gamma$ events. In order that the studied process shows radiation zeros we have to impose the planarity condition. This condition is achieved by requiring that the normal to the plane defined by the beam and the outgoing τ and the plane defined by the photon and the $\bar{\tau}$ directions are approximately parallel. With the notation of Fig 2 this condition reads [4]:

$$|\check{n}_{13}.\check{n}_{1k}| > \cos 20^0, \quad (24)$$

where \check{n}_{13} and \check{n}_{1k} are the normal vector to the plane containing the initial e^- and final τ and the initial e^- and the final γ respectively. For the direction of the lepton τ we have considered a bin centered in $\theta_\tau = \theta_{CM}$ of width 10^0 . In this conditions we integrate over

$$\theta_{CM} - 5^0 < \theta_\tau < \theta_{CM} + 5^0 \quad (25)$$

Although we choose cuts that mimic the soft-photon kinematics we do not expect strict zeros due to the binnig, the planarity cut and the integration over the photon energy. In this conditions, close to the zeros positions as indicated in Fig. 3 , we expect dips in the photon distribution.

In Figs. 5-10 we show the θ_γ distribution for different values of θ_{CM} and for different values of the center of mass energy. In this plots we include the contribution from the effective interactions for different values of $\epsilon = \epsilon_\gamma = \epsilon_Z$. The results of this approximated calculation shows that the effective contribution does not exhibit dips of the same depth as the standard model one. The effect is small at low energy but due the behaviour of the effective coupling the effect grows appreciably with the center of mass energy. The aim of the present work is only show how the anomalous tau lepton couplings destroy the radiation zeros and produce, in principle, an observable effect. Study about the real sensitivity of this effect require a more realistic analysis with realistic detection errors which is beyond the scope of the present work.

V. CONCLUSION

In this work we have investigated the possibility to use radiation zeros in the $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ process to bound anomalous contributions to τ magnetic and weak moment. We have implemented a Monte Carlo program to integrate the phase space including realistic cuts. The anomalous contribution is seen as an apartment from the standard model prediction in the regions where it has radiation zeros. To decide if this kind of observable could be useful it is necessary a most realistic study that takes account of the possible stage of high luminosity in the electron-positron collider.

Acknowledgements

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Figure Captions

Figure 1: Feynman graph contributing to the amplitude of the $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ process.

Figure 2: Parameterisation of the kinematics for the soft photon $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ scattering in the e^+e^- center of mass frame.

Figure 3: Radiation zero position for $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ scattering in the $\theta_{CM}, \theta_\gamma$ plane. The solid (dashed) line represents $\theta_\gamma = \arccos[\mp \frac{1}{\sqrt{2}}\sqrt{1 - \cos\theta_{CM}}]$ respectively.

Figure 4: Amplitude squared, in an arbitrary unity, for the standard model (dashed line) and for the standard model plus the anomalous contribution (solid line) for $\sqrt{s} = 1000\text{GeV}$, $\omega_\gamma = 100\text{GeV}$, $\theta_{CM} = 20^\circ$ and $\epsilon_Z = \epsilon_\gamma = 0.1$

Figure 5: Differential cross-section for the $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ scattering as a function of θ_γ for $\theta_{CM} = 20^\circ$ and $\sqrt{s} = 500\text{GeV}$. Solid step line represents the standard model contribution ($\epsilon_\gamma = \epsilon_Z = \epsilon = 0$). Up triangle and dark circle represent the standard model plus the anomalous contribution for $\epsilon_\gamma = \epsilon_Z = \epsilon = 0.1$ and 0.05 respectively.

Figure 6: Differential cross-section for the $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ scattering as a function of θ_γ for $\theta_{CM} = 90^\circ$ and $\sqrt{s} = 500\text{GeV}$. Solid step line represent the standard model contribution ($\epsilon_\gamma = \epsilon_Z = \epsilon = 0$). Up triangle and dark circle represents the standard model plus the anomalous contribution for $\epsilon_\gamma = \epsilon_Z = \epsilon = 0.1$ and 0.07 respectively.

Figure 7: Differential cross-section for the $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ scattering as a function of θ_γ for $\theta_{CM} = 20^\circ$ and $\sqrt{s} = 1000\text{GeV}$. Solid step line represent the standard model contribution ($\epsilon_\gamma = \epsilon_Z = \epsilon = 0$). Up triangle and dark circle represents the standard model plus the anomalous contribution for $\epsilon_\gamma = \epsilon_Z = \epsilon = 0.1$ and 0.03 respectively.

Figure 8: Differential cross-section for the $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ scattering as a function of θ_γ for $\theta_{CM} = 90^\circ$ and $\sqrt{s} = 1000\text{GeV}$. Solid step line represent the standard model contribution ($\epsilon_\gamma = \epsilon_Z = \epsilon = 0$). Up triangle and dark circle represents the standard model plus the anomalous contribution for $\epsilon_\gamma = \epsilon_Z = \epsilon = 0.1$ and 0.05 respectively.

Figure 9: Differential cross-section for the $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ scattering as a function of θ_γ for $\theta_{CM} = 20^\circ$ and $\sqrt{s} = 2000\text{GeV}$. Solid step line represents the standard model contribution ($\epsilon_\gamma = \epsilon_Z = \epsilon = 0$). Up triangle and dark circle represent the standard model plus the anomalous contribution for $\epsilon_\gamma = \epsilon_Z = \epsilon = 0.1$ and 0.05 respectively.

Figure 10: Differential cross-section for the $e^+e^- \rightarrow \tau\bar{\tau}\gamma$ scattering as a function of θ_γ for $\theta_{CM} = 90^\circ$ and $\sqrt{s} = 2000\text{GeV}$. Solid step line represent the standard model contribution ($\epsilon_\gamma = \epsilon_Z = \epsilon = 0$). Up triangle and dark circle represents the standard model plus the anomalous contribution for $\epsilon_\gamma = \epsilon_Z = \epsilon = 0.06$ and 0.03 respectively.

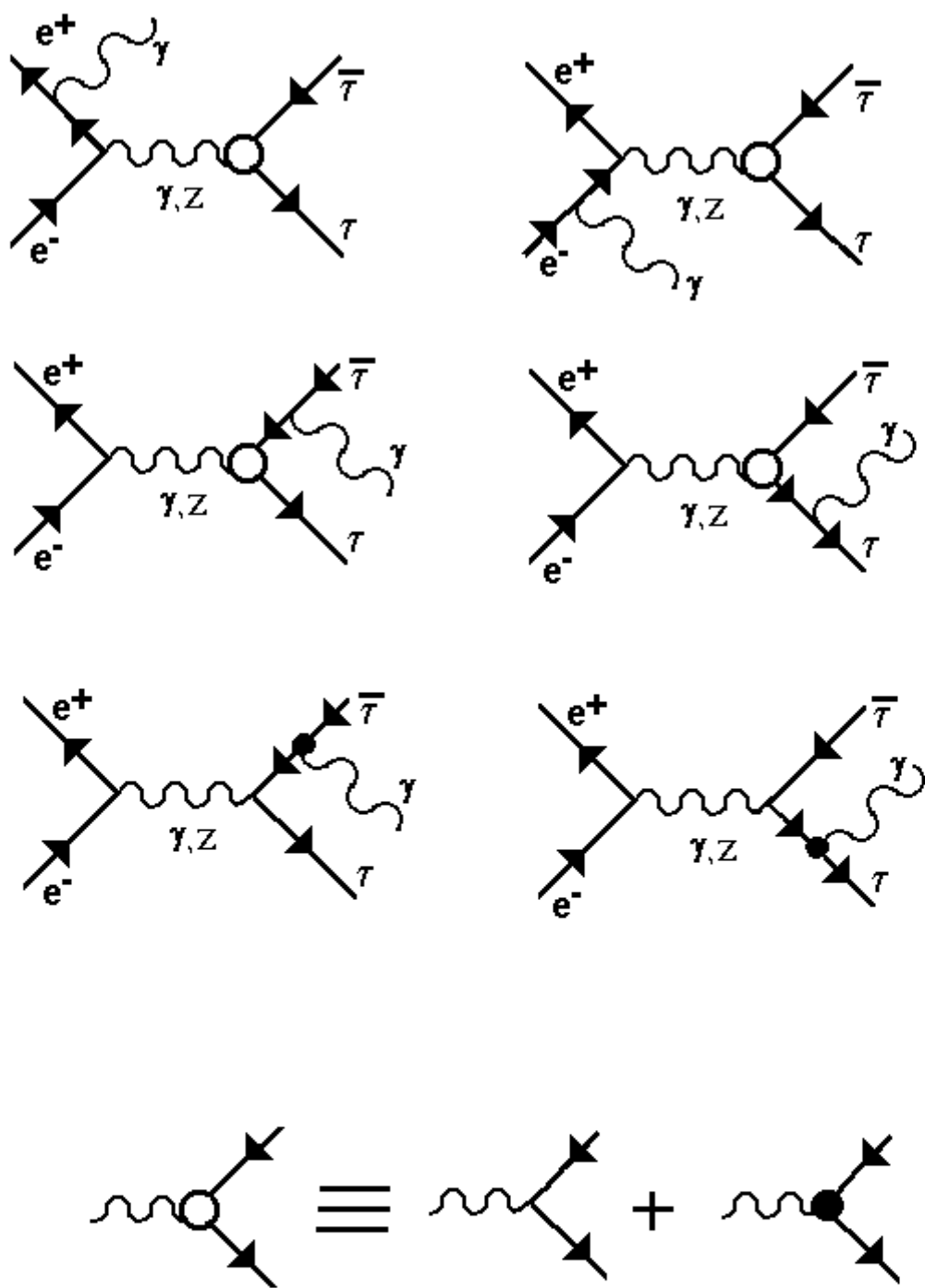


Fig. 1

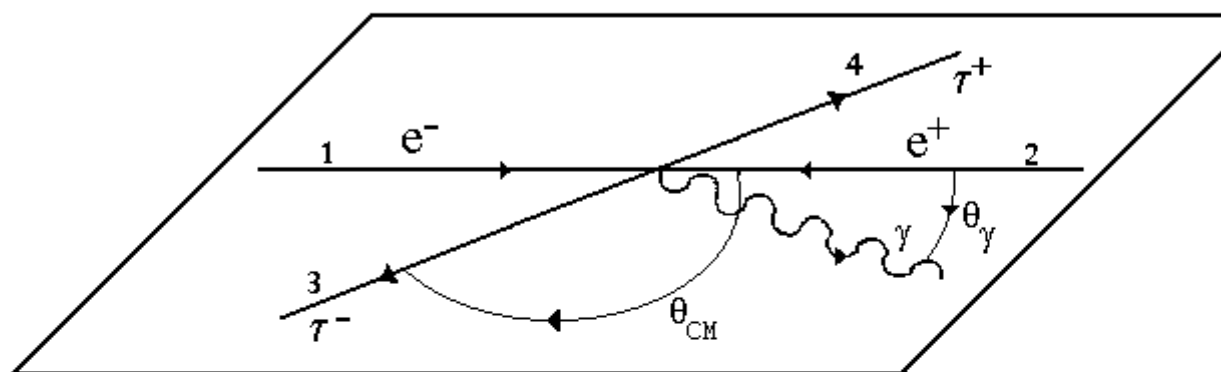


FIG . 2

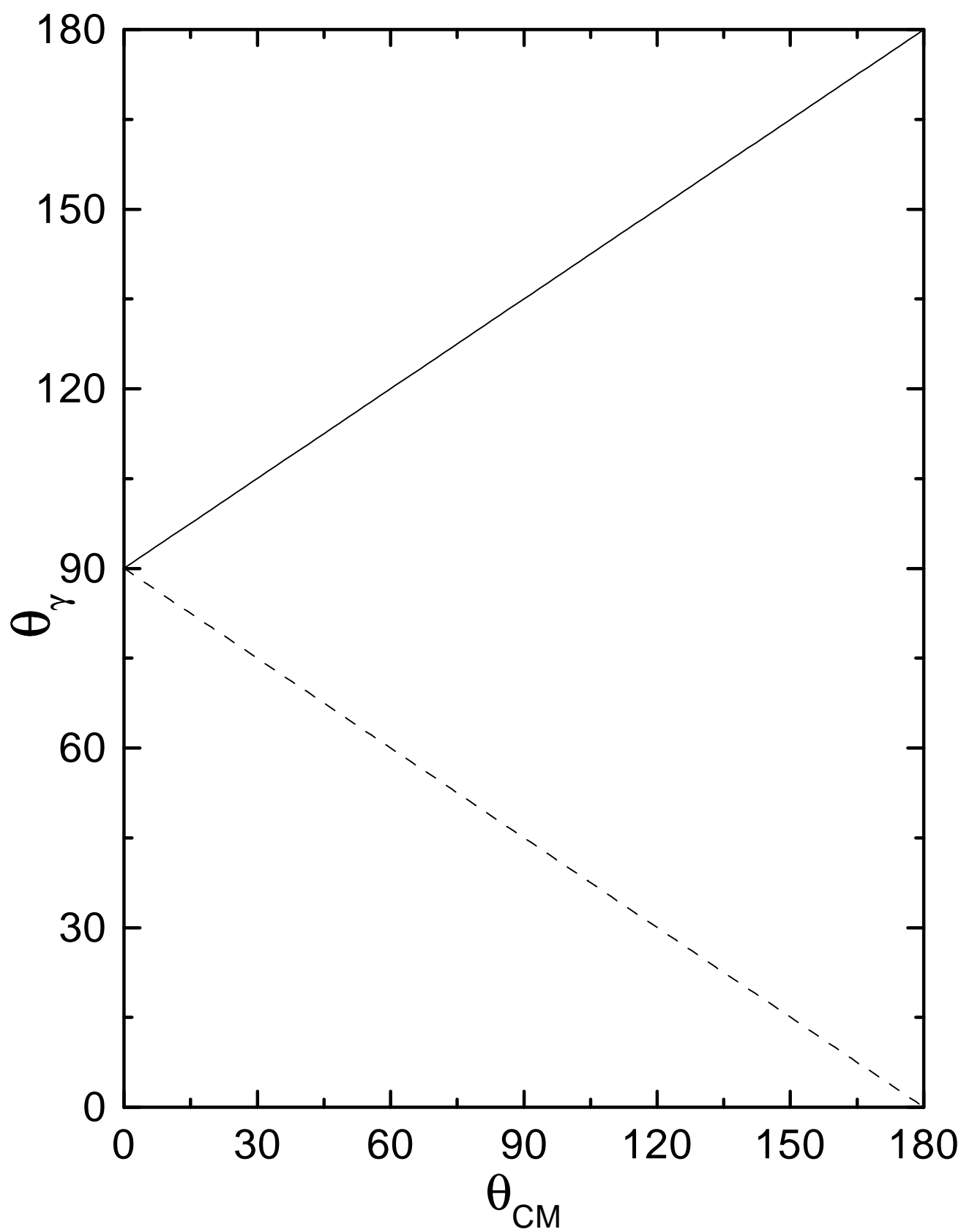


Fig.3

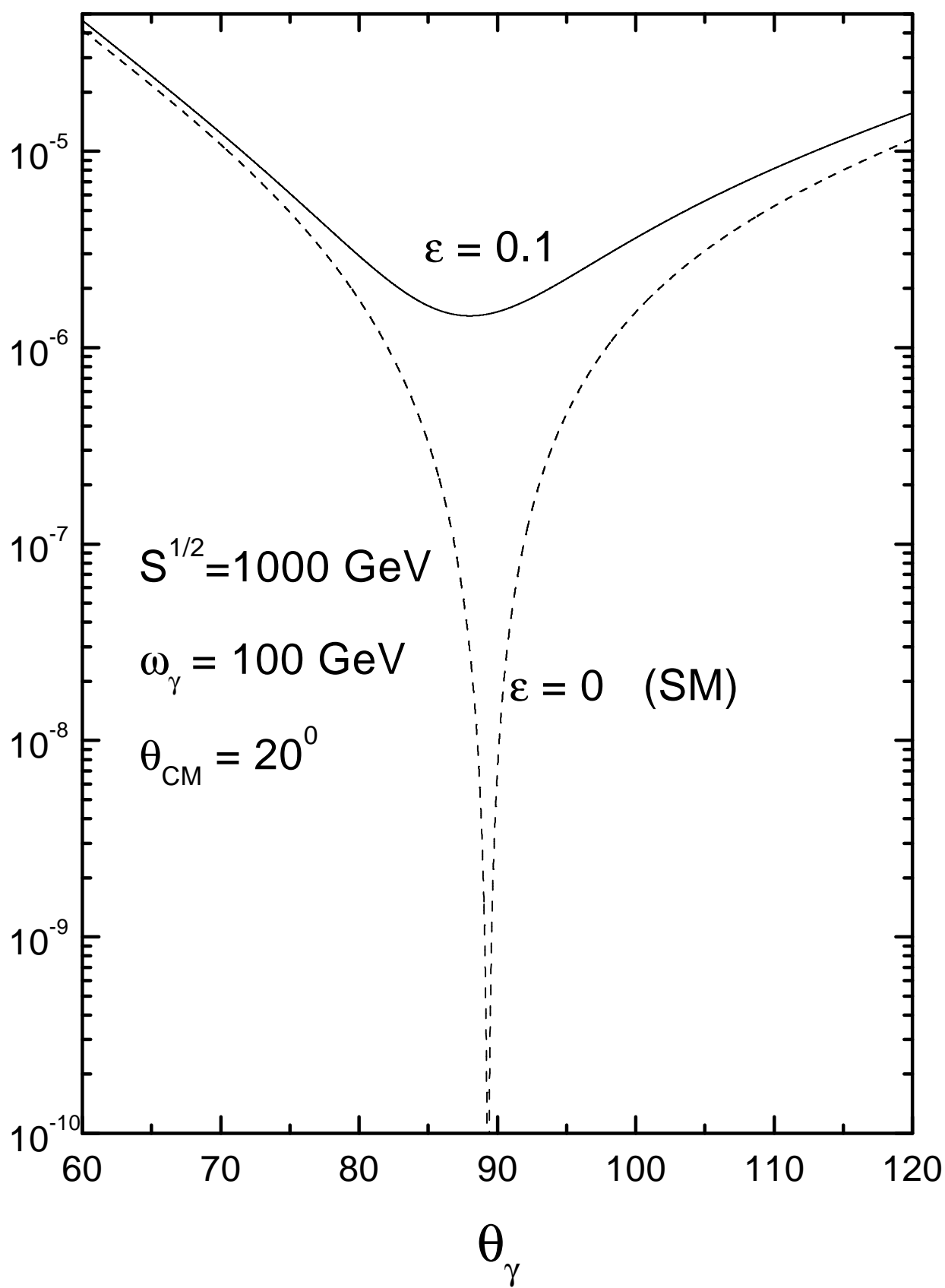


Fig.4

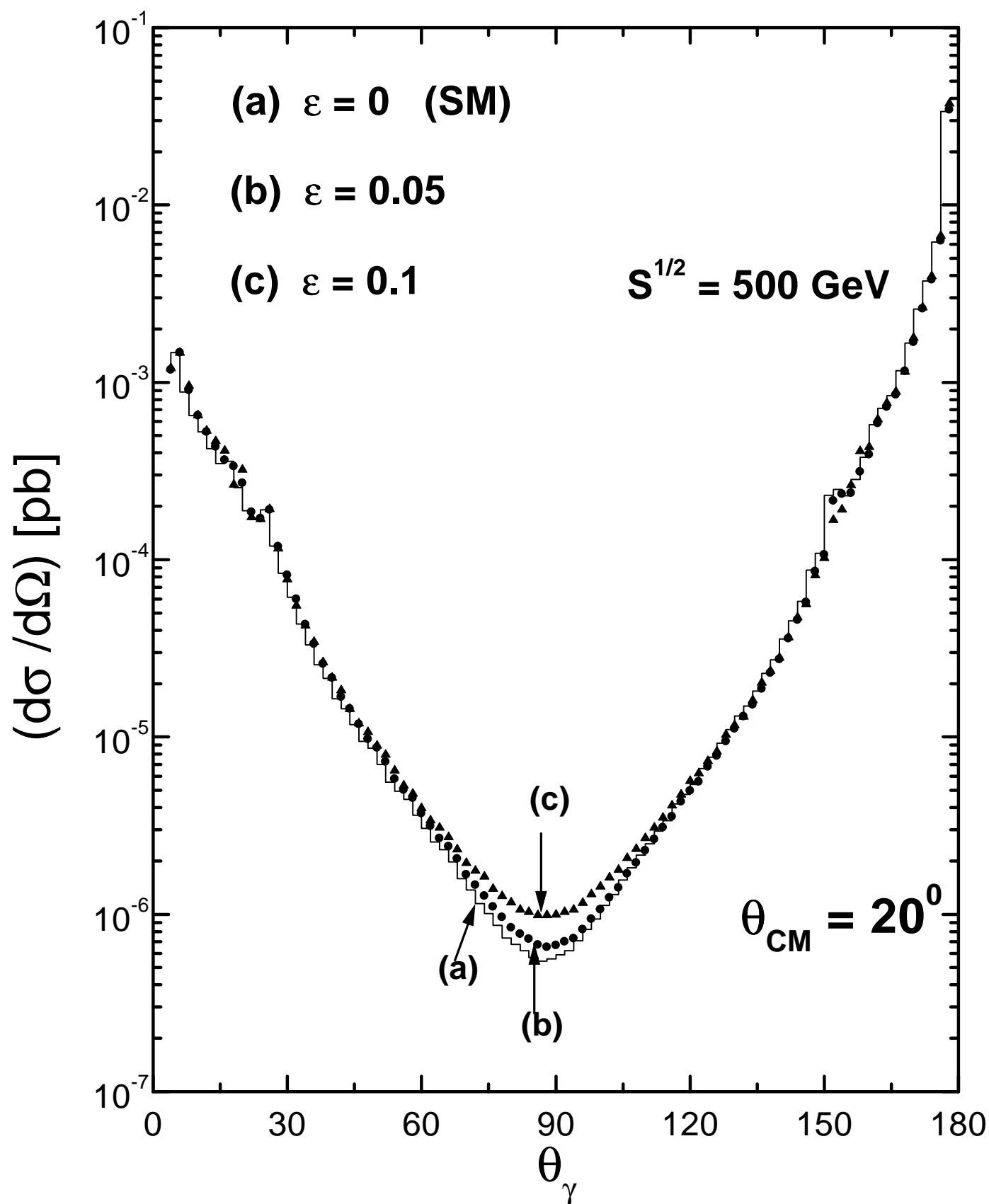


Fig.5

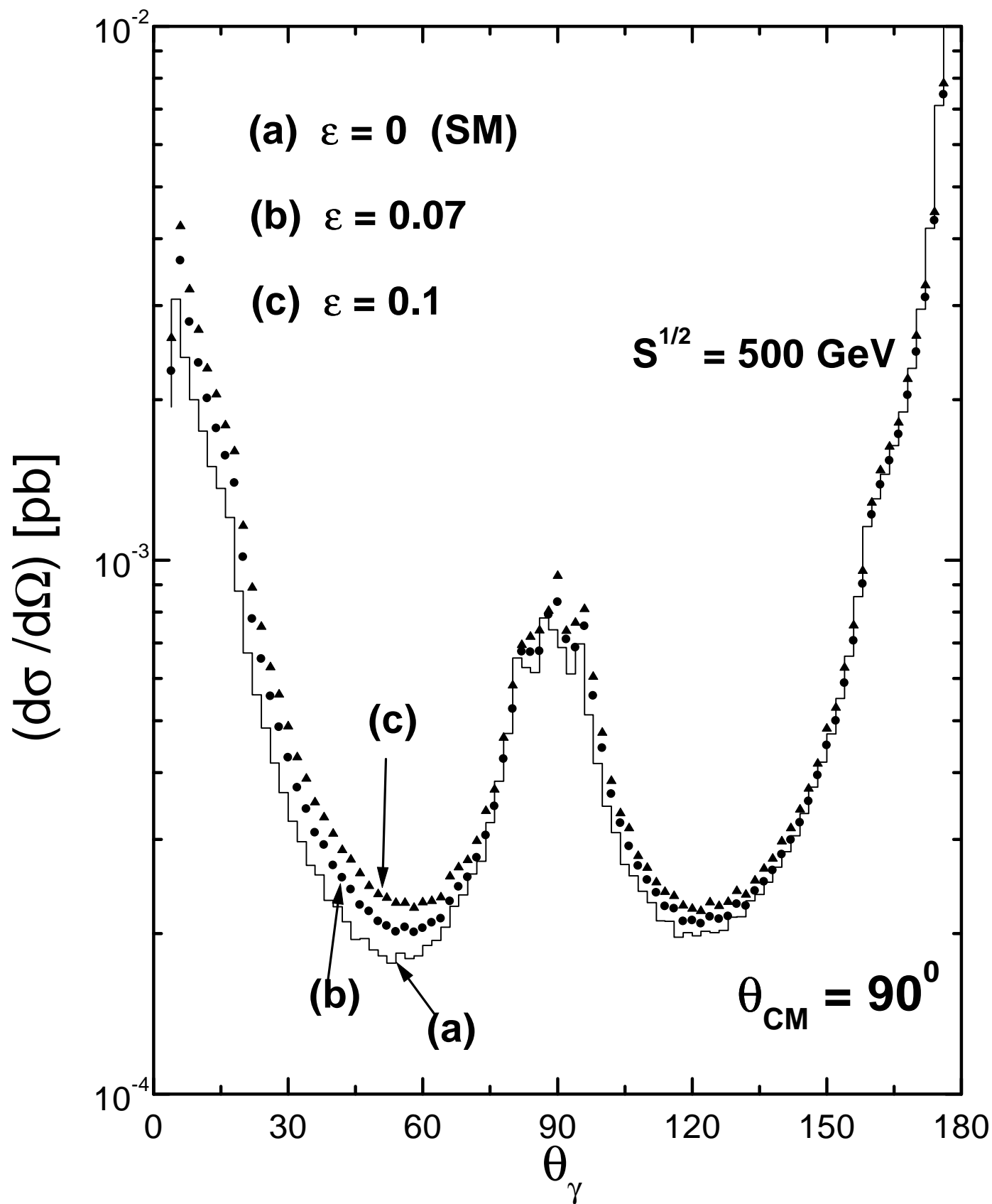


Fig.6

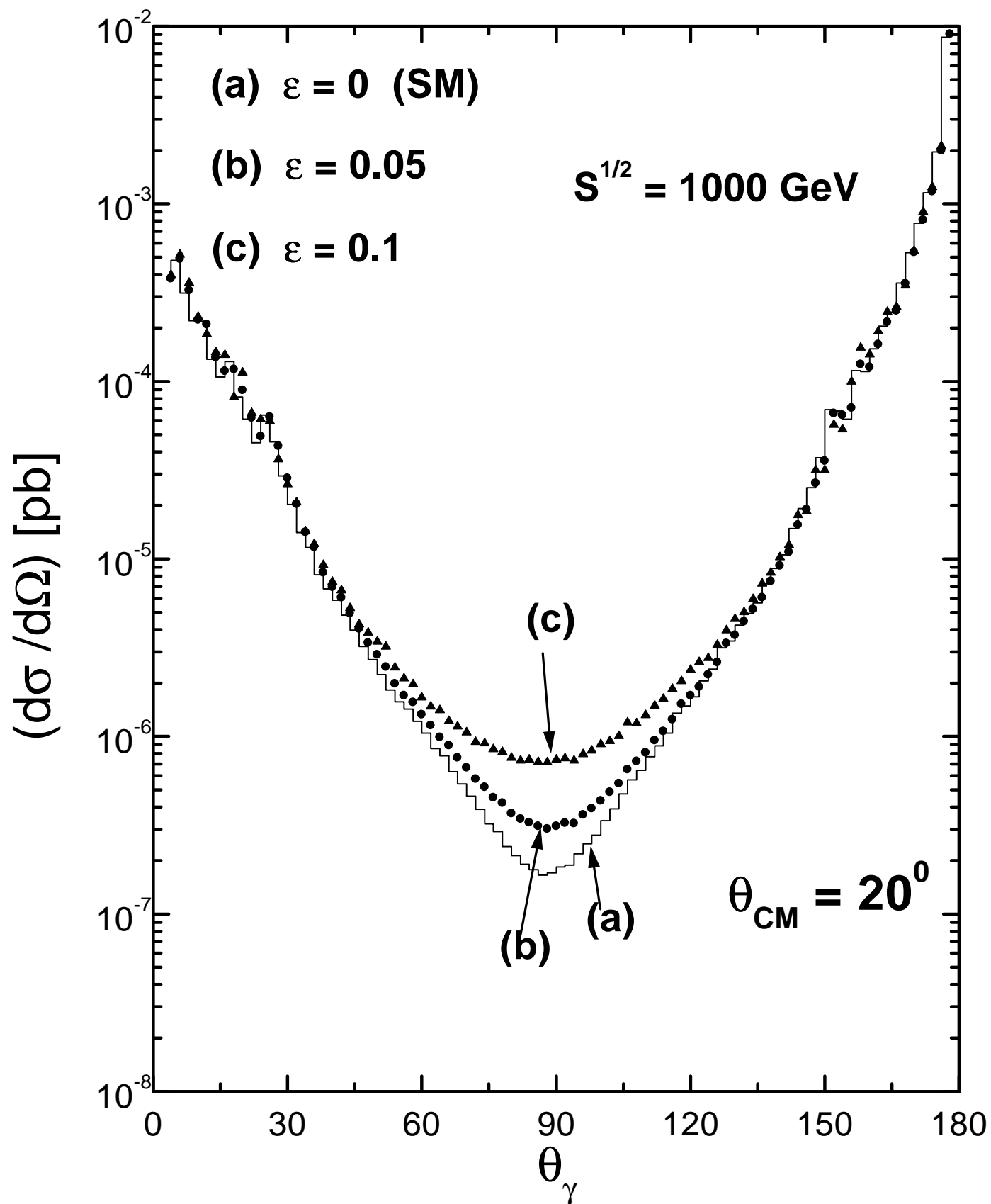


Fig.7

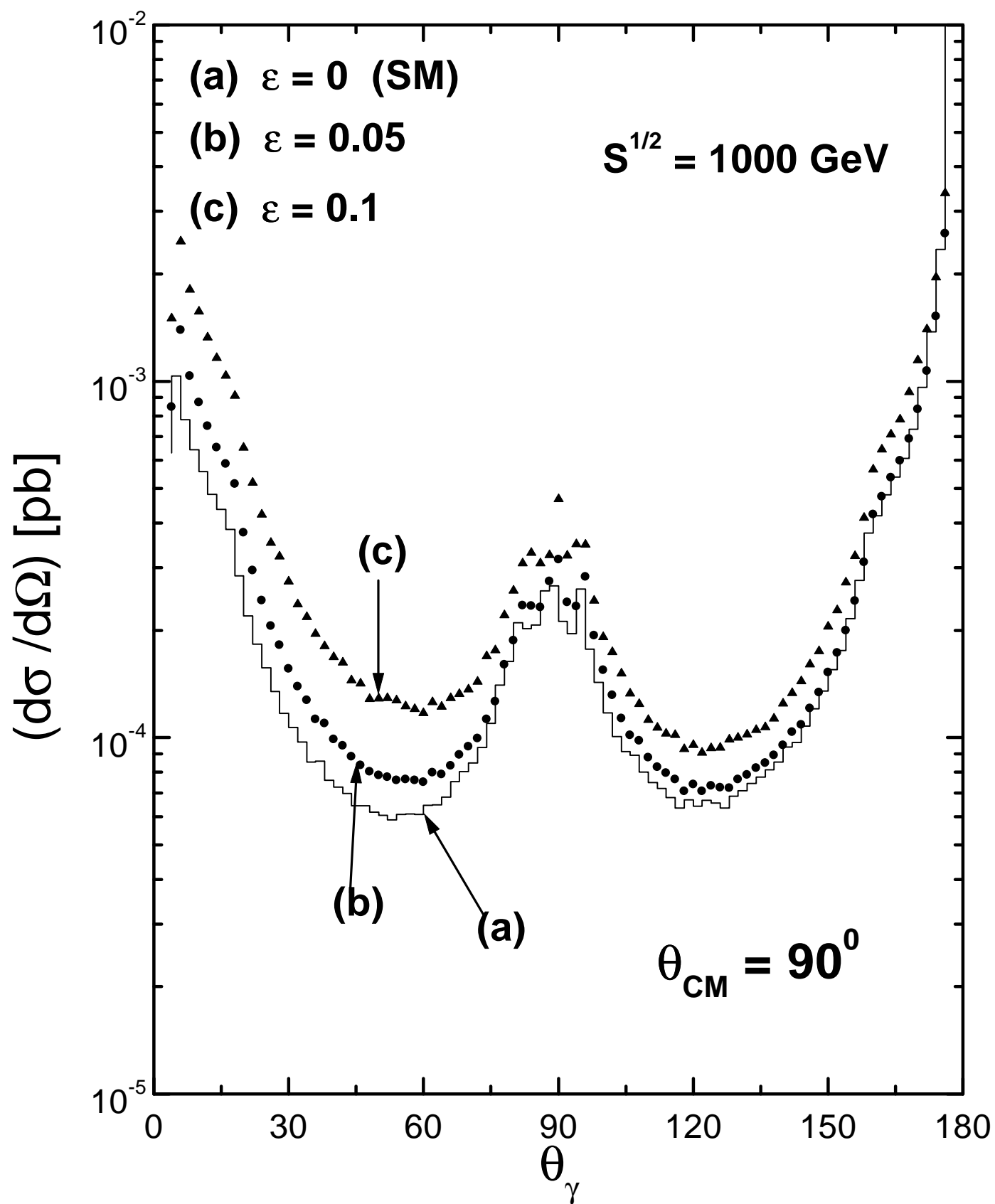


Fig.8

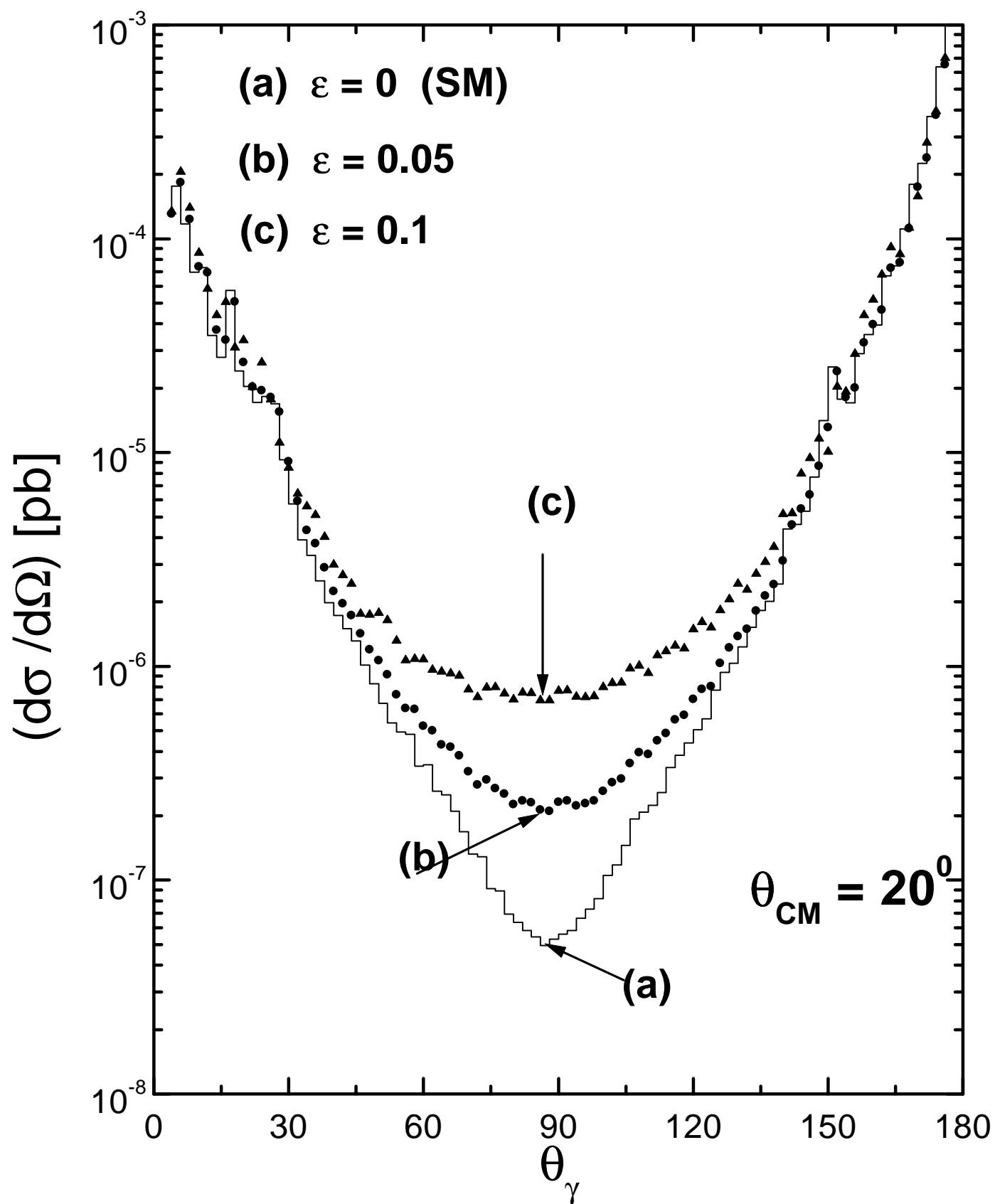


Fig.9

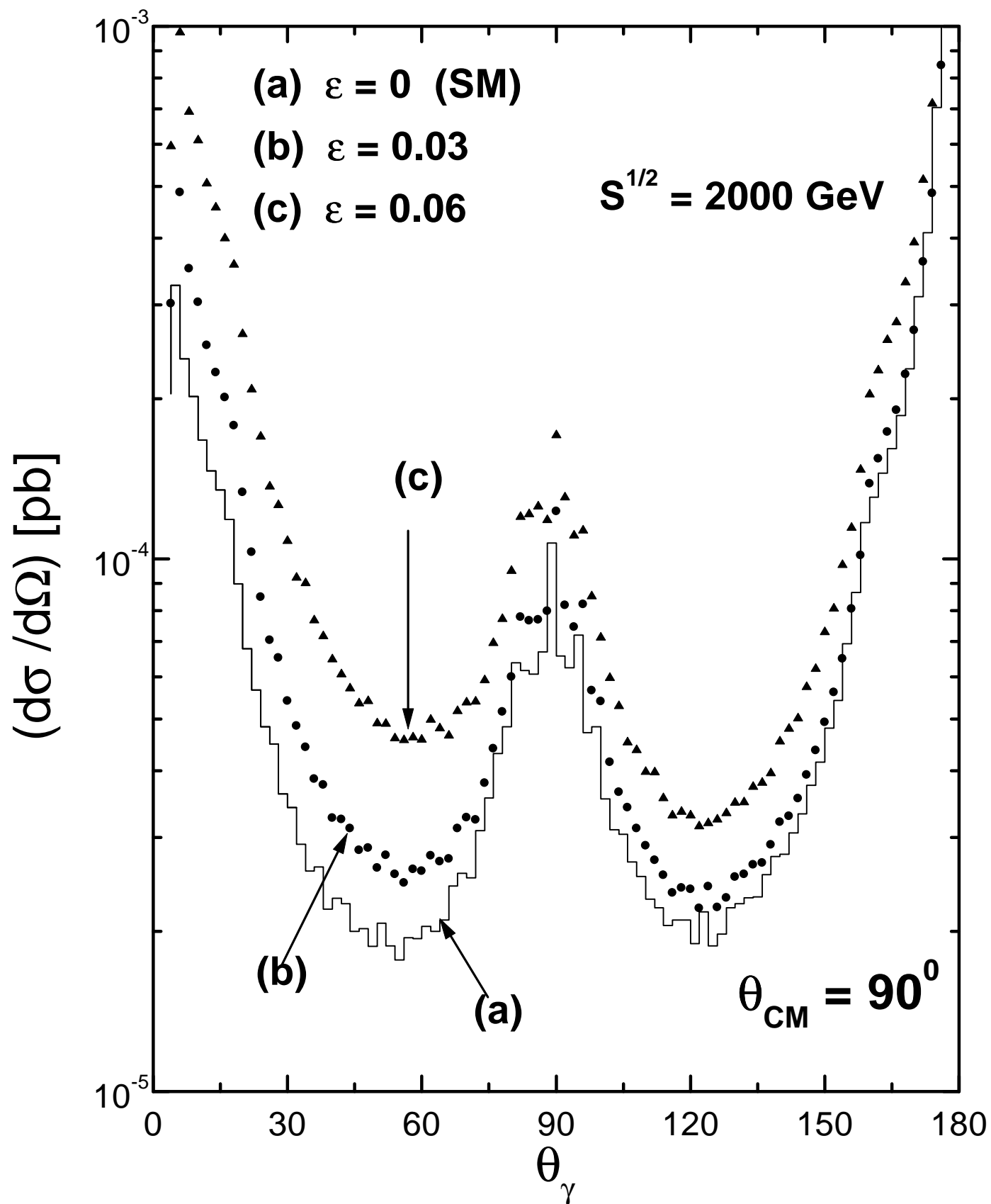


Fig.10